Complex networks are an emerging property of hierarchical preferential attachment

Laurent Hébert-Dufresne, Edward Laurence, Antoine Allard, Jean-Gabriel Young and Louis J. Dubé
Département de Physique, de Génie Physique et d’Optique, Université Laval, Québec, Canada

Summary

Scale independence is observed in all aspects of human life and often modeled through preferential attachment (PA). Network science and PA processes tend to focus on one feature at a time; e.g. degree distribution [1] or community structure [2]. Complex networks are constructs obtained by projecting complex hierarchical systems on a set of nodes and links; collapsing geographical/spatial/cultural/professional correlations. Why not directly model the hierarchical system itself instead of its projection? What can emerge from a simple hierarchy of scale independent organizations?

Hierarchical Preferential Attachment (HPA) features:
- the simplicity of preferential attachment,
- complex networks as an emerging property.

Complex networks emerge from hierarchy?
Hierarchical systems produce networks when projecting under a chosen level of structure. Correlations inter and intra levels of structures dictate properties of the network:
- locally, degree and clustering,
- globally: centrality, self-similarity,
- a complex properties such as geometrical mapping!

Hierarchy makes complex networks complex, HPA is perfectly suited to model scale-independent networks.

Case study: movie production structure

Projection for a realization of HPA:
- Project the system in a network of co-producing credit links between producers who have produced together, regardless of companies and country.
- Random HPA network captures structure from real network not captured by Standard PA:
  1. degree distribution $n(k)$
  2. local clustering coefficient $C(k)$ around nodes of degree $k$ ($C(k) = \frac{k}{k-2}$ in Standard PA)
  3. distribution $n(c)$ of coreness $c$, i.e. number of nodes in a shell of the $k$-core decomposition $n(c) = \delta_{c,k}$ in Standard PA)

Proof of concept: Fractality and geometrical mapping

Fractal (and non-fractal) networks from hierarchy:
- HPA yields fractal and non-fractal networks: self-similarity might imply hierarchy; the opposite is not true:
  - Well-mixed hierarchies have a network diameter $D$ scaling with the logarithm of the number of nodes $N$ (non-fractal)
  - Systems with well defined hierarchy lead to a power-law relation between $D$ and $N$ (fractal)
- Fractality is uncovered with box-counting [3]: groups of nodes within a distance $r$ (number of links) are assigned to the same box. The fractal-dimension $d_f$ relates the number $N_r$ of boxes and their size $r$: $N_r \propto r^{-d_f}$.

Figure on the left: box counting results on a fractal network (protein interaction network of Homo Sapiens) and a non-fractal network (the Internet at the level of autonomous systems) [3].
- HPA models how both of these networks span and cover their respective space.

Hyperbolic mapping of networks [4]:

Mapping of a network: assign geometrical positions to nodes to embed the network in an hyperbolic space. Nodes close (in links) in the network must be geometrically close (in space).
Navigability of complex networks:
- predicts existence of links as a function of geometrical distance between nodes, enabling an efficient navigation.
- is not captured by classical preferential attachment.

Figure on the left: probability of connection $P(l)$ between nodes at a distance $l$ after an inferred projection of the networks onto an hyperbolic space [4]:
- The Internet and its HPA model share a similar scaling exponent for their degree distribution (inset).
- The CCM (Correlated Configuration Model) corresponds to a rewired Internet preserving degree distribution and degree-degree correlations, but obviously lacking the more complex structural correlations.
- Geometrical constraints can emerge simply from hierarchy.

Bibliography and Acknowledgements